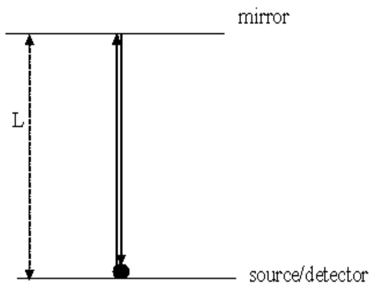
Time Dilation & Length Contraction

TIME DILATION

When traveling at speeds close to the speed of light, time dilates. This means that moving clocks run slow.

Consider a clock aboard a space ship. A light source on the floor projects light onto a mirror on the ceiling.

a) Observed by a person inside the ship

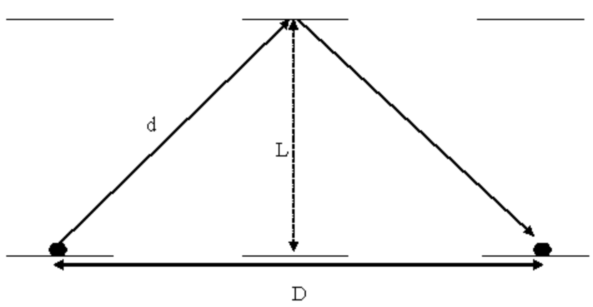


$$t = \frac{\Delta d}{v}$$

$$t_o = \frac{2L}{c}$$

where t_o is the time measured by an observer at rest relative to the event ("proper time")

b) Observed by a person outside the ship



From the diagram:

(1) $D = vt$

(2) $$d = \frac{ct}{2}$$

where D is the distance travelled by the ship

Using the Pythagorean Theorem:

$$\left(\frac{D}{2}\right)^2 + L^2 = d^2$$

Substituting in (1) and (2):

$$\left(\frac{vt}{2}\right)^2 + L^2 = \left(\frac{ct}{2}\right)^2$$

Expanding, we get:

$$\frac{v^2t^2}{4} + L^2 = \frac{c^2t^2}{4}$$

$$v^2t^2 + 4L^2 = c^2t^2$$

Isolate t^2:

$$4L^2 = c^2t^2 - v^2t^2$$
$$4L^2 = t^2(c^2 - v^2)$$
$$t^2 = \frac{4L^2}{c^2 - v^2}$$

Factor c^2 from denominator:

$$t^2 = \frac{4L^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

Square root:

$$t = \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

But $t_o = \frac{2L}{c}$, therefore:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

where t is the time measured by an observer moving at a speed, v, relative to the event ("relativistic time")

This result implies that the time between sending and receiving a light pulse is greater when measured from the Earth then when measured on a moving spaceship.

Consider that the following is a real number.

$$\sqrt{1 - \frac{v^2}{c^2}}$$

Therefore:

$$1 - \frac{v^2}{c^2} > 0$$

$$\frac{v^2}{c^2} < 1$$

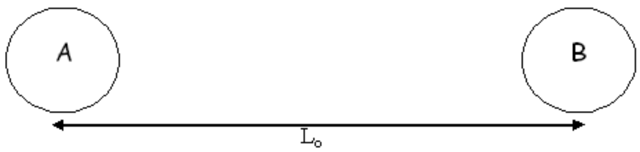
$$\frac{v}{c} < 1$$

$$v < c$$

This means that no material object can have a velocity that is equal to or greater than the speed of light.

LENGTH CONTRACTION

When traveling at speeds close to the speed of light, length contracts. This means that moving objects appear shorter.



L_o is the proper distance between A and B as measured in the frame at rest.

The time for the trip as measured by the stationary observer on the spaceship is t, where

$$t = \frac{L_0}{v}$$

From equation (11), the time for the person on the spaceship is t_o, where

$$t_o = t\sqrt{1 - \frac{v^2}{c^2}}$$

From the frame of reference of the spaceship, the person is at rest therefore A recedes at velocity v and B approaches at speed v.

Therefore,

$$L = vt_o$$

$$L = vt\sqrt{1 - \frac{v^2}{c^2}}$$

But $L_o = vt$,

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}} \qquad (2)$$

where L is the relativistic distance

Examples:

1. What is the mean lifetime of a muon, measured by scientists on Earth, if it is moving at speed o v = 0.70c through the atmosphere? Assume that its lifetime at rest is 2.2 μs.

2. A muon, creating 12km above Earth, travels downward at a speed of 0.98c. Determine the contracted relativistic length the muon experiences as it travels to earth.

Pg 573#2-4(even) Pg 576#5-9(odd) Pg 578#10-12(odd) Pg 579 #1-5 (concepts only)
Pg 579#6-12 (look at . . not for homework) try #9.10 **11.2**